Title
Analysis of a discrete-time single-server queue with bursty imputs for traffic control in ATM networks

Authors
Murata, M.
Oie, Y.
Suda, T.
et al.

Publication Date
1989-05-25

Peer reviewed

Analysis of a Discrete-Time Single-Server Queue with Bursty Inputs for Traffic Control in ATM Networks*

M. Murata† Y. Oie‡ T. Suda†† H. Miyahara†

†Osaka University
Machikaneyama-cho 1-1
Toyonaka, Osaka 560
Japan

‡Sasebo College of Technology
Okishin-machi 1-1
Sasebo, Nagasaki 857-11
Japan

††University of California, Irvine
Irvine
California, CA92717
U.S.A

Technical Report No. 89-13

May 25, 1989

Abstract

Due to a large number of bursty traffic sources that an ATM network is expected to support, controlling network traffic becomes essential to provide a desirable level of network performance with its users. Admission control and traffic smoothing are among the most promising control techniques for an ATM network. To evaluate the performance of an ATM network when it is subject to admission control or traffic smoothing, we build a discrete-time single-server queueing model where a new call joins the existing calls.

In our model, it is assumed that the cell arrivals from a new call follow a general distribution. It is also assumed that the aggregated arrivals of cells from the existing calls form batch arrivals with a general distribution for the batch size and a geometric distribution for the interarrival times of batches. We consider both finite and infinite buffer cases, and analytically obtain the waiting time distribution and cell loss probability for a new call and for existing calls. Our analysis is an exact one. Through numerical examples, we investigate how the network performance depends on the statistics of a new call (burstiness, time that a call stays in active or inactive state, etc.). We also demonstrate the effectiveness of traffic smoothing to reduce network congestion.

---

*This material is based upon work supported by the National Science Foundation under Grant No.DCI-8602052. This research is also in part supported by University of California MICRO Program.

1 Introduction

Broadband packet switching networks offer an attractive way for exchanging multimedia information such as voice, computer data, and images. Among various proposals for implementing broadband networks, asynchronous transfer mode (ATM) seems promising because of its ability to support a broad spectrum of traffic characteristics.

Many of the traffic sources that ATM is expected to support are bursty. For instance, interactive data and compressed video images are considered as bursty. A bursty source may generate cells at a near-peak rate for a while, and a second later, it may become inactive, generating no cells. Due to the dynamic nature of bursty traffic, severe network congestion may occur. Thus, devices to prevent network congestion such as admission control and traffic smoothing techniques are crucial in ATM networks.

In admission control, upon arrival of a new call, a network predicts the performance degradation based on the current network traffic load and traffic statistics of a new call, and accepts the call only when its performance requirement is met. Traffic smoothing is a technique to reduce the burstiness of input traffic to avoid network congestion. The smoothing function could either be performed by an access control at a network-user interface or at a data source by buffering and injecting cells into a network at a slower speed. As in admission control, upon arrival of a new call, smoothing mechanism may or may not be invoked based on the performance prediction.

In both admission control and traffic smoothing, an accurate prediction of network performance becomes important. In most of the past research on the performance of conventional packet switching networks, it has been assumed that input traffic follows a Poisson distribution. This is because most of the traffic found on the conventional packet switching networks is computer-to-computer data, and such data are well-known to follow a Poisson process [1].

Since an ATM network is expected to support a large number of bursty sources with different types of traffic, a Poisson process may no longer be able to describe network traffic accurately.

Therefore, in this paper, we assume an arbitrary arrival process to represent a newly arriving call (along with geometric arrivals for the existing calls on a network), and develop a mathematical model to investigate the effect of input traffic characteristics on the performance of an ATM network.

This paper is organized as follows. In Section 2, we will develop a queueing model for an ATM switch where a number of calls are multiplexed onto an output port of the switch. We will assume that the switch is fast enough so that queueing of cells occurs only on the output. As for the capacity of output buffer, we consider two cases: infinite and finite capacity.

In Section 3, we obtain the z-transform for the waiting time distribution of cells, assuming infinite buffer capacity on the output. In Section 4, assuming finite buffer on the output, we obtain the loss probability of cells and the distribution of queue length on an output.

In Section 5, numerical results are provided to discuss the effects of traffic characteristics on the performance of a switch. We will use an Interrupted Poisson Process (IPP) [2] for the cell generation process from a new call. Section 6 provides concluding remarks.

2 Analytic Model

In this paper, we assume a non-blocking switch fabric. We also assume that a switch is fast enough to handle cells even when all the input ports inject the maximum number of cells into the switch. Thus, buffering is only needed on the outputs [3, 4]. As for the capacity of the buffer at an output port, we consider both infinite and finite cases. We further assume constant size cells. Channel time is slotted, with slot size being equal to a cell transmission time. All the input lines are synchronized. Cell arrivals are considered to occur at slot boundaries.

We focus on one output port of a switch, and consider the case of adding a new call to a set of existing calls on the output port. We analyze the performance of a switch and investigate how the switch performance depends on the traffic characteristics of a new call. We assume that the interarrival time between cells from a new call forms a general distribution with its density

$a(k)$ = Prob[interarrival time = k slots] and z-transform $A(z)$. Generation of cells from a new call is referred to as GI-stream (cell arrival stream with generally and independently distributed interarrival times).

Aggregated arrivals of cells from the existing calls form a Bernoulli process with batch arrivals. In other words, cells arrive in batches, and the batch size (the number of arrivals in a slot) follows a general distribution with the z-transform $B(z)$. Cell arrivals in different slots are independent. This cell arrival process is referred to as M-stream (cell arrival stream with a Markovian interarrival time distribution).

Our traffic model for the existing calls is farely general. For instance, if we assume that each of the existing N calls follows a geometric arrival process with the probabilities p for having an arrival in a slot and $1-p$ for not having an arrival, the sum of these N calls becomes a Bernoulli process with batch arrivals, where the z-transform for the batch size distribution is given by

$$B(z) = \left(1 - \frac{p}{N} + z\frac{p}{N}\right)^N . \tag{1}$$

Using Kendall's notation, our analytical model is denoted as a discrete-time $GI + M^{[X]}/D/1/K$ ($K < \infty$ or $K = \infty$) queue system. Analytical results on this queueing system are not known to date. The closest model so far investigated is a continuous-time queueing system with exponential service time and aggregated arrivals of GI and Poisson processes ($GI + M/M/1/\infty$) [5, 6]. In the following section, we analyze infinite buffer case. The finite buffer case is analyzed in Section 4.

3 Infinite Buffer System

In this section, we consider the infinite buffer case and obtain the z-transform for the waiting time distribution for each of the GI- and M-streams.

3.1 Derivation of Queue Length Distribution

We will observe an output buffer at arrival instances of cells from GI-stream, and relate the queue length distribution at one observation point (say, at $(n+1)$-st arrival from GI-stream) to that of the previous observation point (at n-th arrival from GI-stream). For this purpose, we first obtain the queue length distribution in k slots following the n-th arrival from GI-stream.

Let C_0^n be the random variable representing the queue length immediately before the n-th arrival of a cell from GI-stream, and C_k^n be the random variable representing the queue length at the end of the k-th slot following the n-th arrival of a cell from GI-stream. (See Fig.1.) Assume B is the random variable for the batch size, i.e., the number of cells arriving in a slot from M-stream, and let $b(i)$ be its density, i.e., $b(i) = \text{Prob}[B = i]$.

At the end of the slot where the n-th arrival from GI-stream happens, there are $C_0^n + B + 1 - 1$ cells in the buffer. (C_0^n number of cells were in the buffer immediately before the arrival from GI-stream, B cells from M-stream and one cell from GI-stream join the queue, and one cell is taken for service from a queue.) Thus, we have

$$C_1^n = C_0^n + B. \tag{2}$$

Here, we have assumed the *come right in* strategy [7], i.e., a cell finding the buffer empty is immediately served and removed from the buffer.

Noting that there are no arrivals from GI-stream in the k-th $(k \geq 2)$ slot following the arrival from GI-stream, we have the following recurrence equation for the number of cells (C_k^n) in the buffer:

$$C_k^n = \max(C_{k-1}^n + B - 1, 0), \quad k = 2, 3, \cdots. \tag{3}$$

We assume that the steady state exist for C_k^n and C_0^n. Namely,

$$\lim_{n \to \infty} C_k^n = C_k, \quad k = 1, 2, \cdots, \tag{4}$$

$$\lim_{n \to \infty} C_0^n = C_0. \tag{5}$$

We define $C_k(z)$ and $C_0(z)$ as the z-transforms for C_k and C_0, respectively.

By letting n go to infinity in eqs.(2) and (3), we can obtain the z-transforms for the queue length distributions as follows:

$$C_1(z) = C_0(z)\, B(z), \tag{6}$$

$$C_k(z) = \frac{B(z)}{z}\, C_{k-1}(z) + \frac{z-1}{z}\, C_{k-1}(0)\, B(0), \qquad k = 2, 3, \cdots. \tag{7}$$

Note that $B(z)$ has been previously defined as the z-transform for B.

Applying eq.(7) recursively, and finally using eq.(6), we obtain $C_k(z)$ as a function of the initial queue length distribution $C_{0(z)}$ as follows.

$$C_k(z) = \left(\frac{B(z)}{z}\right)^{k-1} C_0(z)\, B(z) + \sum_{m=1}^{k-1} \left(\frac{B(z)}{z}\right)^{m-1} \frac{z-1}{z}\, C_{k-m}(0)\, B(0), \qquad k = 1, 2, \cdots. \tag{8}$$

$C_0(z)$ in the above equation is easily determined in the following way. With the probability $a(k)$, an arrival of a new cell from GI-stream occurs in k slots following the last arrival from GI-stream. Upon arrival, the new cell finds the queue length distribution $C_k(z)$. Therefore, $C_0(z)$ is given by

$$C_0(z) = \sum_{k=1}^{\infty} a(k)\, C_k(z). \tag{9}$$

Substituting eq.(8) into eq.(9), $C_0(z)$ becomes

$$C_0(z) = z\, C_0(z)\, A\left(\frac{B(z)}{z}\right) + \frac{z-1}{B(z)}\, B(0) \sum_{m=1}^{\infty} \left(\frac{B(z)}{z}\right)^m x(m), \tag{10}$$

where $A(z)$ has been previously defined as the z-transform for the interarrival times of GI-stream cells, and the unknown constants, $x(m)$, are defined as

$$x(m) \triangleq \sum_{k=0}^{\infty} a(m+k)\, C_k(0), \qquad m \geq 1. \tag{11}$$

Noting that the z-transform for the sequence $x(m)$ is given by

$$X(z) = \sum_{m=1}^{\infty} x(m)\, z^m, \tag{12}$$

it is easy to see that the term $\sum_{m=1}^{\infty}(\frac{B(z)}{z})^m x(m)$ in eq.(10) becomes $X(\frac{B(z)}{z})$. Therefore, eq.(10) becomes

$$C_0(z) = z\, C_0(z)\, A(\frac{B(z)}{z}) + \frac{z-1}{z}\, B(0)\, X(\frac{B(z)}{z}). \tag{13}$$

By solving eq.(13), we finally have

$$C_0(z) = \frac{(z-1)\, B(0)\, X(\frac{B(z)}{z})}{\{1 - z\, A(\frac{B(z)}{z})\}\, B(z)}. \tag{14}$$

To obtain the unknown function $X(\cdot)$ in the above equation, we follow the method described in [8]. Appendix summarizes this method.

3.2 Derivation of Waiting Time Distributions for M-Stream and GI-Stream

In this section, we obtain the waiting time distribution for both M-stream and GI-stream. As a rule, in order to select a cell for service from the buffer, we assume random selection among the cells arriving in the same slot, while FCFS is assumed among the cells arriving in different slots.

We first obtain the waiting time distribution for GI-stream. The waiting time for GI-stream cell, say a test GI-cell, consists of two elements; the time to serve all the cells found in the buffer upon arrival of a test GI-cell, and the time to serve the M-stream cells which arrive with a test GI-cell and are served before the test GI-cell. The first element is easily obtained, since we know that the queue length distribution immediately before an arrival of a GI-stream cell is given by $C_0(z)$ [see eq.(14)]. Noting that the service time of a cell is unit time, $C_0(z)$ for the queue length also gives the z-transform for the time to serve all the cells found in the buffer upon arrival of a test GI-cell.

The second element, the time spent in serving M-stream cells before a test GI-cell, is obtained in the following way. With the probability $b(i-1)$, a total of i cells ($i-1$ M-stream cells and a test GI-cell) arrive in a slot. Among these i cells, a test GI-cell is placed at the j-th position with the probability $1/i$ ($i \geq j$). Therefore, the probability that $j-1$ M-stream cells are served prior to a test GI-cell is given by

$$d_{GI}(j) = \sum_{i=j}^{\infty} \frac{b(i-1)}{i}, \qquad j = 1, 2, \cdots. \tag{15}$$

The z-transform for $d_{GI}(j)$ becomes

$$\begin{aligned} D_{GI}(z) &\triangleq \sum_{j=1}^{\infty} d_{GI}(j)\, z^j \\ &= \sum_{j=1}^{\infty} z^j \sum_{i=j}^{\infty} \frac{b(i-1)}{i} \\ &= \sum_{i=1}^{\infty} \{ \sum_{j=1}^{i} \frac{b(i-1)}{i}\, z^j \} \\ &= \sum_{i=1}^{\infty} \frac{b(i-1)}{i} \, \frac{z\,(1-z^i)}{1-z} \\ &= \frac{z}{1-z} \int_z^1 B(x)dx. \end{aligned} \tag{16}$$

Since the service time of a cell is unit time, $D_{GI}(z)$ for the number of cells served prior to the test GI-cell gives the z-transform for the time to serve the M-stream cells which arrive at the same time as the test GI-cell and are served before the test GI-cell.

Using $C_0(z)$ and $D_{GI}(z)$, we find the z-transform for the waiting time distribution of the GI-stream cells, $W_{GI}(z)$, becomes

$$W_{GI}(z) = C_0(z)\, D_{GI}(z). \tag{17}$$

Note that the waiting time for a test GI-cell includes its own service time. The mean waiting time is then obtained as

$$EW_{GI} = W_{GI}^{(1)}(1), \tag{18}$$

where $W_{GI}^{(1)}$ is the first derivative of $W_{GI}(z)$.

Next, we derive the waiting time distribution for M-stream. Let us assume that a test cell from M-stream (a test M-cell) arrives in the k-th slot following the last arrival of a GI-stream cell. The waiting time for a test M-cell consists of two elements. The first element, the time to serve the cells in the queue found upon arrival of a test M-cell, is obtained from $C_{k-1}(z)$, the z-transform for the queue length distribution immediately before the arrival of a test M-cell.

The second element of the waiting time for a test M-cell is the amount of time it takes before it is selected for service out of the cells arrived with it. First, assume that a test M-cell arrives with the GI-stream cell (i.e., $k = 1$). Let $b_0(i)$ be the conditional probability of having i cell arrivals from M-stream in a slot, given that one or more M-stream cells arrive. Note that $b_0(i)$ is given by $b(i)/(1 - b(0))$. Then, $b_0(i-1)$ gives the probability of having a total of i cell arrivals in a slot ($i-1$ M-stream cells and a GI-stream cell). Among these i cells, a test M-cell is placed at the j-th position with the probability $1/i$ ($i \geq j$). Noting that there are $\binom{i-1}{1}$ possible ways to choose a test M-cell from $i-1$ M-stream cells, and following the similar discussion to obtain eq.(15), the density for the waiting times of M-stream cells is obtained as

$$d_{M,1}(j) = \frac{1}{G} \sum_{i=j}^{\infty} (i-1) \frac{b_0(i-1)}{i}, \tag{19}$$

where G is a normalizing constant and is equal to the mean of $b_0(i)$, $EB_0 = \sum_{i=1}^{\infty} i b_0(i)$. We also considered the fact that the service time is unit time to obtain eq.(19). After some manipulation, the z-transform for $d_{M,1}(j)$, $D_{M,1}(z)$, is obtained as

$$D_{M,1}(z) = \frac{1}{EB_0} \frac{z}{1-z} [1 - zB_0(z) - \int_z^1 B_0(x)dx]. \tag{20}$$

Next, assume that the arrival of a test M-cell occurs in the k-th slot ($k \geq 2$) following the last arrival of a GI-stream cell. Note that, in this case, there are no arrivals of a GI-stream cell in that slot. Using the result in [9], the z-transform for the time until a test M-cell is selected becomes

$$\begin{aligned} D_{M,k}(z) &= \frac{1}{EB_0} \frac{z}{1-z} (1 - B_0(z)), \quad k = 2, 3, \cdots, \\ &\triangleq D_M(z). \end{aligned} \tag{21}$$

Using the probability $s(k)$ that the arrival of a test M-cell falls in the k-th slot following the last arrival of a GI-stream cell, we have the z-transform for the waiting time distribution of M-stream cells as follows:

$$W_M(z) = \sum_{k=1}^{\infty} s(k)\, C_{k-1}(z)\, D_{M,k}(z), \tag{22}$$

where $s(k)$ is given by

$$s(k) = \sum_{i=k}^{\infty} \frac{a(i)}{EA}, \qquad k = 1, 2, \cdots. \tag{23}$$

Note that $EA \triangleq \sum_{k=1}^{\infty} k a(k)$ is the average interarrival time of GI-stream cells.

By expanding eq.(22), we have

$$W_M(z) = s(1)\, C_0(z)\, D_{M,1}(z) + s(2)\, C_1(z)\, D_M(z) + D_M(z) \sum_{k=3}^{\infty} s(k)\, C_{k-1}(z). \tag{24}$$

The only unknown terms in the above equation are $C_k(z)$s in the term $\sum_{k=3}^{\infty} s(k) C_{k-1}(z)$. This term $\sum_{k=3}^{\infty} s(k) C_{k-1}(z)$ becomes

$$\begin{aligned}
&\sum_{k=3}^{\infty} s(k)\, C_{k-1}(z) \\
&= \sum_{k=3}^{\infty} s(k) \left\{ \left(\frac{B(z)}{z}\right)^{k-2} C_0(z)\, B(z) + \sum_{m=1}^{k-2} \left(\frac{B(z)}{z}\right)^{m-1} C_{k-1-m}(0)\, B(0) \right\} \\
&= \frac{z^2\, C_0(z)}{B(z)}\, S\!\left(\frac{B(z)}{z}\right) - s(1)\, z\, C_0(z) - s(2)\, B(z)\, C_0(z) \\
&\qquad + \frac{z(z-1)}{B(z)^2}\, B(0) \sum_{m=2}^{\infty} \left(\frac{B(z)}{z}\right)^m \sum_{k=1}^{\infty} C_k(0)\, s(m+k),
\end{aligned} \tag{25}$$

where $S(z) \triangleq \sum_{k=1}^{\infty} s(k)\, z^k$ is the z-transform for the sequence, $s(k)$, and is

$$S(z) = \frac{1}{EA} \frac{z\,(1 - A(z))}{(1 - z)}. \tag{26}$$

We define the last summation term in eq.(25) as $g(m)$, i.e.,

$$g(m) \triangleq \sum_{k=1}^{\infty} C_k(0)\, s(m+k), \qquad m = 1, 2, \cdots. \tag{27}$$

Then,

$$\begin{aligned}
g(m) &= \sum_{k=1}^{\infty} C_k(0)\, s(m+k) \\
&= \sum_{k=1}^{\infty} C_k(0) \sum_{n=m+k}^{\infty} \frac{a(n)}{EA} \\
&= \frac{1}{EA} \sum_{i=m}^{\infty} \sum_{k=1}^{\infty} C_k(0)\, a(i+k) \\
&= \frac{1}{EA} \sum_{i=m}^{\infty} x(i).
\end{aligned} \tag{28}$$

To obtain the last equation, we used eq.(11). The z-transform for the sequence $g(m)$, $G(z)$, then becomes

$$\begin{aligned} G(z) &= \sum_{m=1}^{\infty} g(m)\, z^m \\ &= \frac{1}{EA} \sum_{m=1}^{\infty} \sum_{i=m}^{\infty} x(i)\, z^m \\ &= \frac{1}{EA} \frac{z}{1-z} (X(1) - X(z)). \end{aligned} \tag{29}$$

From eqs.(27) and (29), the term $\sum_{m=2}^{\infty} \left(\frac{B(z)}{z}\right)^m \sum_{k=1}^{\infty} C_k(0) s(m+k)$ in the eq.(25) can be represented by

$$\begin{aligned} \sum_{m=2}^{\infty} \left(\frac{B(z)}{z}\right)^m \sum_{k=1}^{\infty} C_k(0)\, s(m+k) &= \sum_{m=2}^{\infty} \left(\frac{B(z)}{z}\right)^m g(m) \\ &= G(\frac{B(z)}{z}) - \frac{B(z)}{z} g(1) \\ &= \frac{1}{EA} \frac{B(z)}{z - B(z)} \{X(1) - X(\frac{B(z)}{z})\} - \frac{1}{EA} \frac{B(z)}{z}. \end{aligned} \tag{30}$$

Substituting eqs.(26) and (30) into eq.(25), and then using eq.(22), we finally obtain

$$\begin{aligned} W_M(z) = \frac{1}{EA} \Bigg\{ & C_0(z)\, (D_{M,1}(z) - z\, D_M(z)) + B(0)\, \frac{z-1}{B(z)}\, D_M(z) \\ & + B(0)\, \frac{z\,(z-1)}{(z - B(z))B(z)} \left[X(1) - X(\frac{B(z)}{z}) \right] \Bigg\}. \end{aligned} \tag{31}$$

4 Finite Buffer System

In this section, we analyze a finite buffer system and obtain the distribution for queue length and loss probability of cells for each of the M-stream and GI-stream. It is assumed that the buffer can hold a maximum of K cells. Throughout this section, we assume a general distribution for cell arrivals from GI-stream with the restriction that the maximum interarrival time is finite. We also assume that the number of cells arriving in a slot from M-stream (batch size) follows a general distribution and has the finite maximum value.

4.1 Derivation of Queue Length Distribution

First, we define the following operators [10, 11]:

$$\pi^m(y(j)) = \begin{cases} y(j) & j < m \\ \sum_{i=m}^{\infty} y(i) & j = m \\ 0 & j > m \end{cases} \tag{32}$$

$$\pi_0(y(j)) = \begin{cases} 0 & j < 0 \\ \sum_{i=-\infty}^{0} y(i) & j = 0 \\ y(j) & j > 0 \end{cases} \tag{33}$$

A convolution operator for two discrete distributions is denoted by a symbol $\star$ as

$$y_1(j) \star y_2(j) \triangleq \sum_{i=-\infty}^{\infty} y_1(j-i) * y_2(i). \tag{34}$$

Let $\overline{C_k^n}$ be a random variable representing the system length (i.e., the number of cells in the buffer plus a cell being served at a server) observed at the beginning of the k-th slot following the n-th arrival of a cell from GI-stream. (For instance, $\overline{C_1^n}$ is the system length immediately after the n-th arrival of GI-stream cell; $\overline{C_2^n}$ is the system length at the beginning of the second slot; and so on. See Fig.1.)

The $\overline{C_1^n}$ cells in the system (the buffer and the server combined) at the beginning of the first slot, which followed the arrival of a GI-stream cell, are those present at the buffer before the arrival of the GI-stream cell (C_0^n cells), a newly arrived GI-stream cell (1 cell), and newly arrived M-stream cells (B cells). Note that the cell being served in the previous slot leaves the system immediately before the arrival of a GI-stream cell, and thus, need not to be counted. Thus, we have

$$\overline{C_1^n} \quad = \quad \min(C_0^n + 1 + B, K + 1). \tag{35}$$

Since $\overline{C_1^n}$ (system length) is equal to C_1^n (queue length) plus one, if there is a cell being served, we have

$$C_1^n = \max(\overline{C_1^n} - 1, 0). \tag{36}$$

From the similar discussion above, we have the following recurrence relations for $k = 2, 3, \cdots$:

$$\overline{C_k^n} = \min(C_{k-1}^n + B, K + 1), \quad k = 2, 3, \cdots. \tag{37}$$

$$C_k^n = \max(\overline{C_k^n} - 1, 0), \quad k = 2, 3, \cdots, \tag{38}$$

Let $c_k^n(j)$ and $\overline{c}_k^n(j)$ be the density function for C_k^n and $\overline{C}_k^n$, respectively. Namely, $c_k^n(j) = \text{Prob}[C_k^n = j]$ $(k = 0, 1, \cdots)$ and $\overline{c}_k^n(j) = \text{Prob}[\overline{C_k^n} = j]$ $(k = 1, 2, \cdots)$. $b(i)$ is the density of B, batch size of M-stream arrivals. (See Section 3.1 for the definition of $b(i)$.) Using π-operations on eqs.(35) through (38), we have

$$\overline{c}_1^n(j) = \pi^{K+1}(c_0^n(j-1) \star b(j-1)), \quad 1 \leq j \leq K + 1, \tag{39}$$

$$c_i^n(j) = \pi_0(\overline{c}_i^n(j+1)), \quad 0 \leq j \leq K, \quad i = 1, 2, \cdots, \tag{40}$$

$$\overline{c}_i^n(j) = \pi^{K+1}(c_{i-1}^n(j) \star b(j)), \quad 0 \leq j \leq K + 1, \quad i = 2, 3, \cdots. \tag{41}$$

From the similar discussion to derive eq.(9), $\{c_0^{n+1}(j), 0 \leq j \leq K\}$, the queue length distribution immediately before the $(n+1)$-st arrival of a GI-stream cell, is given as a function of $c_k^n(j)$ by

$$c_0^{n+1}(j) = \sum_{k=1}^{\infty} a(k)\, c_k^n(j), \quad 0 \leq j \leq K. \tag{42}$$

We can calculate the values of $c_0^{n+1}(j)(0 \leq j \leq K)$ from eqs.(39) through (42) by following the computational diagram illustrated in Fig.2.

The steady state probability that there are j cells in the buffer immediately before an arrival of a GI-stream cell is given by

$$c_0(j) = \lim_{n \to \infty} c_0^n(j), \quad 0 \leq j \leq K. \tag{43}$$

If distributions for the interarrival times of GI-stream and for the batch size of M-stream are identically and independently distributed, we can apply the computational procedure in Fig.2 repeatedly to obtain the steady state probabilities $c_0(j)(0 \leq j \leq K)$.

4.2 Derivation of Loss Probabilities for M-Stream and GI-Stream

We first consider $P_{GI}(j)$, the conditional cell loss probability for GI-stream given that the queue length (C_0) immediately before an arrival of a GI-stream cell is j. Assume that i cells arrive from M-stream along with a GI-stream cell and find j cells in the buffer, i.e., $C_0 = j$. if $i+1+j > K+1$, $i+j-K$ cells are lost due to buffer overflow. Thus, the probability that a GI-stream cell is among the lost cells is given by $(i+j-K)/(i+1)$. Since the probability of having i arrivals from M-stream is $b(i)$, the conditional probability, $P_{GI}(j)$, then becomes

$$P_{GI}(j) = \sum_{i=K-j+1}^{\infty} \frac{i+j-K}{i+1} b(i). \tag{44}$$

By unconditioning $P_{GI}(j)$ on the values of $c_0 = j$, the cell loss probability for GI-stream becomes

$$\begin{aligned} P_{GI} &= \sum_{j=0}^{K} c_0(j)\, P_{GI}(j) \\ &= \sum_{j=0}^{K} c_0(j) \sum_{i=K-j+1}^{\infty} \frac{i+j-K}{i+1} b(i). \end{aligned} \tag{45}$$

In the following, we will obtain the cell loss probability for an M-stream. First, we assume that i cells arrive from M-stream along with a GI-stream cell and find j cells in the buffer. We focus on one of the i M-stream cells, a test M-cell, and obtain the probability that a test M-cell is lost. If $i+1+j > K+1$, $i+j-K$ cells are lost due to buffer overflow, and $(i+1)-(i+j-K) = 1-j+K$ cells are accepted into the buffer. If a GI-stream cell is one of the $i-j+K$ accepted cells (probability of this is $(1-j+K)/(i+1)$), the probability that a test M-cell is lost becomes $(i+j-K)/i$. If a GI-stream cell is one of the $i+j-K$ lost cells (probability of this is $(i+j-K)/(i+1)$), the probability that a test M-cell is lost becomes $(i+j-K-1)/i$. Since the probability that a test M-cell is contained in a batch of size i is $ib_0(i)/EB_0$ [9], the cell loss probability $P_{M,1}(j)$ for M-stream in the first slot following the arrival of a GI-stream cell is given by

$$\begin{aligned} P_{M,1}(j) &= \sum_{i=K-j+1}^{\infty} \frac{i\, b_0(i)}{EB_0} \left[\frac{1-j+K}{i+1} \frac{i+j-K}{i} + \frac{i+j-K}{i+1} \frac{i+j-K-1}{i} \right] \\ &= \sum_{i=K-j+1}^{\infty} \frac{i\, b_0(i)}{EB_0} \frac{i+j-K}{i+1}. \end{aligned}$$

Next, we consider the case where i cells arrive from M-stream in the k-th slot ($k \geq 2$) following an arrival of a GI-stream cell and find k cells in the buffer. In this case, there are no arrivals from GI-stream. If $i + j > K + 1$, $i + j - K - 1$ cells are lost due to buffer overflow. The probability that a test M-cell is lost is $(i + j - K - 1)/i$.

Therefore, $P_{M,k}(j)$, the cell loss probability at k-th slot ($k \geq 2$) following an arrival of a GI-stream cell given that there are j cells in the buffer upon arrival of a test M-cell, becomes

$$P_{M,k}(j) = \sum_{i=K-j+2}^{\infty} \frac{j + i - K - 1}{EB_0} b_0(i), \quad 0 \leq j \leq K, \quad k = 2, 3, \cdots. \tag{46}$$

Since the probability that the queue length is j upon arrival of a test M-cell is $c_k(j)$, and that the probability that the arrival of a test M-cell falls in the k-th slot following the last arrival of a GI-stream cell is $s(k)$ [see eq.(23)], the cell loss probability for M-stream becomes

$$P_M = \sum_{k=1}^{\infty} s(k) \sum_{j=0}^{K} c_k(j)\, P_{M,k}(j). \tag{47}$$

5 Numerical Discussions

5.1 Traffic Model for ATM Networks

Although our analysis allows arbitrary arrival distributions for GI-stream, in the following numerical examples, we assume a discrete-time Interrupted Poisson Process (IPP) for GI-stream. While IPP in the continuous-time domain was originally introduced by Kuczura[2] for an overflow process, it has been recently considered to be suitable to describe bursty traffic in ATM networks (see, for example, [12]). A discrete-time IPP is a process with the following features: (1) IPP is a process with two states, *ON* and *OFF*, which appear in turn, (2) IPP changes from ON to OFF with probability $1 - \gamma$ per slot, from OFF to ON with probability $1 - \omega$ per slot. The IPP stays in ON state (OFF state) for geometrically distributed length of time with the average $1/(1-\gamma)$ $[1/(1-\omega)]$. (3) IPP generates a cell with the rate λ per slot only when it is in ON state, and does not generate any cells when in OFF state. (See Fig.3.)

Interarrival times of cells from GI-stream form IPP. The z-transform and its moments for IPP can be easily obtained in a similar approach used to develop for a continuous time IPP [2], and given by the following. The z-transform, $A_{IPP}(z)$, is

$$A_{IPP}(z) = \frac{\lambda\{(1-\gamma-\omega)z^2 + \gamma z\}}{1-(\gamma(1-\lambda)+\omega)z-(1-\lambda)(1-\gamma-\omega)z^2}, \tag{48}$$

and the first and second moments, denoted by $M^{(1)}_{IPP}$ and $M^{(2)}_{IPP}$, respectively, are given by

$$M^{(1)}_{IPP} = \frac{1/(1-\gamma)+1/(1-\omega)}{\lambda/(1-\gamma)}, \tag{49}$$

$$M^{(2)}_{IPP} = \frac{\left\{\begin{array}{l} 2\lambda(1-\omega)(1-\gamma-\omega)-2(-\lambda\gamma-(2-\gamma-\omega) \\ +2\lambda(1-\omega))(2-\gamma-\omega) \end{array}\right\}}{\lambda^2(1-\omega)^2} + M^{(1)}_{IPP}. \tag{50}$$

The variance of IPP, VA_{IPP}, then becomes

$$VA_{IPP} = \frac{\sqrt{M^{(2)}_{IPP} - [M^{(1)}_{IPP}]^2}}{M^{(1)}_{IPP}}. \tag{51}$$

As for M-stream, we assume geometric arrivals of cells in batches with a Poisson distribution for the batch size. The z-transform for the batch size is given by

$$B(z) = e^{-\lambda_M(1-z)}, \tag{52}$$

where λ_M is the average batch size. Since we take the transmission time of a cell as unit time, λ_M is same as the traffic load (ρ_M).

5.2 Effects of Burstiness of Traffic on Performance

In Figs.4.1, 4.2 and 4.3, the value of λ is assumed to be 1.0; namely, GI-stream generates a cell in each and every slot during ON-state (see Fig.3). We will change the values of γ and ω, keeping the ratio of the average ON length $[1/(1-\gamma)]$ and the average OFF length $[1/(1-\omega)]$ constant 1/10. In this case, the traffic load of GI-stream $\rho_{GI} = 1/M^{(1)}_{IPP} = \frac{\lambda/(1-\gamma)}{1/(1-\gamma)+1/(1-\omega)}$ becomes 1/11. In the following, we examine the effects of different ON and OFF lengths on the performance.

Fig.4.1 shows the mean waiting times of cells as a function of ρ_M (the traffic load of M-stream) for the cases of ($\gamma = 0.9$, $\omega = 0.99$) and ($\gamma = 0.99$, $\omega = 0.999$). The average ON and OFF lengths are 10 slots and 100 slots, when $\gamma = 0.9$ and $\omega = 0.99$, and 100 slots and 1000 slots, when $\gamma = 0.99$ and $\omega = 0.999$. Infinite buffer capacity is assumed in Fig.4.1. Real lines illustrate the mean waiting times for GI-stream. Whereas, the dotted lines represent waiting times for M-stream. For comparison purposes, we also included the mean waiting time for the case where there is only M-stream (this case is indicated by "M-stream only" in the figure.) This figure shows that the mean waiting times for GI-stream and M-stream in the case of (ON length, OFF length) = (100, 1000) are much larger than those in the case of (ON length, OFF length) = (100, 1000). By comparing the mean waiting times for M-stream for different cases, it is apparent that, if the average ON and OFF lengths of GI-stream are longer, an increase in the M-stream waiting time is greater for the same value of ρ_M.

For the infinite buffer system, we analytically obtained the waiting time distributions for GI-stream and M-stream in Section 3. Fig.4.2 shows, as a function of ρ_M, the 99.9% cumulative waiting time, i.e., the value of the minimum integer T which satisfies Prob[waiting time $\leq T$] ≥ 0.999. We have assumed the same parameter values as in Fig.4.1. This figure confirms the observation made in Fig.4.1: the performance diminishes proportionally to the length of the ON and OFF lengths of GI-stream. Thus, the longer the ON and OFF lengths of GI-stream are, the worse the performance becomes.

Figs.4.3 and 4.4 are for the finite buffer system, and show the cell loss probability as a function of ρ_M (traffic load of M-stream). Buffer size K is the parameter in both figures. In Fig.4.3, the average ON and OFF lengths are 10 and 100 slots, respectively, and in Fig.4.4, the average ON and OFF lengths are 100 and 1000 slots, respectively. From these two figures, we observe that the cell loss probabilities are larger, when the ON and OFF lengths of GI-stream are longer. For instance, compare the cell loss probabilities for GI-stream for $K = 50$ in Fig.4.3 and in Fig.4.4. If the cell loss of less than 10^{-6} should be satisfied, ρ_M has to be less than approximately 0.4 in Fig.4.3, while this cell loss requirement cannot be satisfied at all in Fig.4.4.

Note that the interarrival times of GI-stream cells must be bounded in order for our analysis of a finite buffer system to be applicable. On the other hand, in this numerical example section, we have assumed that the GI-stream arrival process follows an IPP, where infinitely long interarrival times are possible. To obtain figures for the finite buffer system in this section, as an approximation, we truncate the tail of the IPP distribution at far distance and sweep the probability in the truncated tail onto the truncated point.

It is often difficult for a network to acquire complete statistics of input traffic, and therefore, a network may be forced to make a decision based on incomplete information. For instance, the statistics of the arrival process of a new call may be unknown to a network except for its average interarrival time, and the network may have to decide if it can accommodate a new call without causing severe network congestion. In such a case, a network may assume some well-known processes (such as a geometric process) as an arrival process and may make performance prediction. The following question immediately arises: how accurate is the prediction based on incomplete information on input traffic? To answer this question, we have calculated various performance measures assuming that GI-stream follows a geometric process, and examined how accurately (or inaccurately) this predicts the performance of the system where, in fact, GI-stream follows IPP. Curves indicated as "GIM-stream" in Fig.4.1 through Fig.4.4 (and in Fig.5.1 through Fig.5.4) show the performance measures for the case that GI-stream follows a geometric arrival process. The average interarrival time of a geometric process is set to be the same as that of the IPP arrival process in these figures.

By comparing the mean waiting time for the geometric arrival case (indicated as "GIM-stream") against to those for the IPP arrival case (indicated as "GI-stream" and "M-stream") in Fig.4.1, we can see that the differences in the waiting times for these cases are rather large. Furthermore, the longer the ON and OFF lengths of GI-stream are, the greater the differences are. This same trend is also found in Figs.4.2, 4.3 and 4.4. Note that, even when the traffic load of M-stream (ρ_M) is small, the performance for the geometric arrival case is far from that for the IPP arrival case. (This is clearly shown in Figs.4.3 and 4.4.) Assuming a geometric arrival process does not provide a good performance estimate, if the arrival process, in fact, is IPP.

5.3 Effects of Traffic Smoothing on Performance

In the following four figures (Fig.5.1 through Fig.5.4), we assume traffic smoothing on GI-stream and examine the effects of traffic smoothing on the performance of a network. The purpose of traffic smoothing[1] is to throttle cell inputs into a network in order to avoid burst cell transmissions. The smoothing function could either be performed by an access control at a network-user interface or at a data source by buffering and injecting cells into a network at a slower speed. When a network is congested by momentary bursts of traffic, traffic smoothing reduces network congestion by suppressing inputs to the network. As a result, a network with traffic smoothing may be able to support a greater number of calls than a network without one.

In the following, to model traffic smoothing, we change the average ON length $[1/(1-\gamma)]$ of GI-stream, keeping the traffic load $\rho_{GI} = \frac{\lambda/(1-\gamma)}{1/(1-\gamma)+1/(1-\omega)}$ of GI-stream constant. When the average ON length is longer, cell arrivals are spread over a longer time period, giving smoother (less bursty) GI-stream traffic. We further assume that the sum of the average ON and OFF lengths $[1/(1-\gamma)+1/(1-\omega)]$ of GI-stream is constant (c). The value of c is assumed to be 100 slots in Figs.5.1, 5.2 and 5.3, and 1000 slots in Fig.5.4. ρ_{GI} (traffic load of GI-stream) is 0.1 in all the figures.

In Figs.5.1 and 5.2, we illustrate the effects of smoothing GI-stream traffic on the waiting times of M-stream. Fig.5.1 shows the mean waiting times, and Fig.5.2 shows 99.9% cumulative waiting

[1] Traffic smoothing is sometimes referred to as shaping.

times. Buffer size (K) is assumed to be infinite. In these figures, the horizontal line represents the average ON length $[1/(1-\gamma)]$ of GI-stream. Both the arrival rate per slot λ and the average ON length are changed to keep the traffic load $\rho_{GI} = 0.1$. Two levels of the traffic load of M-stream are considered: $\rho_M = 0.5$ and $\rho_M = 0.8$. In both figures, it is clearly shown that traffic smoothing achieves better performance for M-stream. As the GI-stream traffic becomes smoother (i.e., as the value of $1/(1-\gamma)$ becomes larger), both the mean waiting time (in Fig.5.1) and 99.9% cumulative waiting time (in Fig.5.2) for M-stream become smaller.

Figs.5.1 and 5.2 also show that both the mean waiting time and cumulative waiting time for GI-stream become smaller as the average ON length increases. This is because GI-stream becomes less bursty as the average ON length becomes longer.

Fig.5.3 assumes finite buffer capacity and shows the cell loss probability as a function of the average ON length. The sum of the average ON and OFF length of GI-stream is assumed to be 100 slots. Fig.5.4 shows the mean waiting times for the case where $c = 1000$ and $\lambda/(1-\gamma) = 100$. In Fig.5.4, buffer capacity (K) is assumed to be infinite. As we have seen in Figs.5.1 and 5.2, as the GI-stream traffic becomes smoother, the waiting times of M-stream become smaller. From our observation, it may be concluded that smoothing GI-stream traffic has a significant effect on improving M-stream performance.

6 Concluding Remarks

In this paper, we have provided an exact analysis for a discrete-time single-server queueing system where two different arrival processes are assumed: (1) arrivals of cells with a general interarrival time distribution, and (2) geometric arrivals of cells in batches with a general batch size distribution. We have considered both infinite and finite buffer capacity cases. Our model clearly describes a switch in an ATM network, when it is subject to call admission control.

Through numerical examples, we investigated how the network performance depends on the degree of burstiness of the input traffic. We have also observed that smoothing input traffic reduces network congestion.

Our analytical approach for the finite buffer system in Section 4 follows a technique widely used in a signal processing area, and requires iterations and, at each iteration, the convolution operations are used. The computational cost may be large in some cases. We believe, however, that the following may save computational costs: (1) Efficient algorithms like FFT [13] may be employed to save computing costs of calculating convolution. (2) The queue length distribution, $c_0(j)$, for the infinite buffer system may be used as the initial values in iterations. This achieves a quick convergence.

References

[1] L.Kleinrock, *Queueing Systems*, Vol.2, John Wiley and Sons Pub., 1976.

[2] A. Kuczura, "The Interrupted Poisson Process as an Overflow Process," *The Bell System Technical Journal*, Vol.52, No.3, pp.437-448, March 1973.

[3] M.J.Karol, M.J.Hluchyj, and S.P.Morgan, "Input versus Output Queueing on a Space-Division Packet Switch," *IEEE Transactions on Communications*, Vol.COM-35, No.11, pp.1347-1356, December 1987.

[4] Y. Oie, M. Murata, K. Kubota, and H. Miyahara, "Effect of Speedup in Nonblocking Packet Switch," to appear in the Proceedings of *IEEE ICC'89.*

[5] A. Kuczura, "Queues with Mixed Renewal and Poisson Inputs," *The Bell System Technical Journal*, Vol.51, No.6, pp.1305-1326, July-August, 1972.

[6] A. Kuczura, "Loss Systems with Mixed Renewal and Poisson Inputs," *Operations Research*, Vol.51, No.6, pp.787-795, July-August, 1972.

[7] J.F. Hayes, *Modeling and Analysis of Computer Communications Networks*, Plenum Press, New York, 1984.

[8] H. Bruneel, "Analysis of Discrete-Time Buffers with One Single Output Channel, Subjected to a General Interruption Process," *Performance '84*, E.Gelenbe (Editor), pp.103-115, 1984.

[9] P.J. Burke, "Delays in Single-Server Queues with Batch Input," *Operations Research*, Vol.51, No.6, pp.830-833, July-August, 1975.

[10] P. Tran-Gia, "Discrete-Time Analysis for the Interdeparture Distribution of $GI/G/1$ Queues," *Proceedings of Teletraffic Analysis and Computer Performance Evaluation*, O.J.Boxma,

J.W.Cohen H.C.Tijms (Editors), pp.341-357, Elsevier Science Publishers (North-Holland), 1986.

[11] P. Tran-Gia and H. Ahmadi, "Analysis of a Discrete-Time $G^{[X]}/D/1$-S Queueing System with Application in Packet-Switching Systems," *Proceedings of IEEE INFOCOM'88*, pp.9A.1.1-1.10, 1988.

[12] J.Y. Hui, "Resource Allocation for Broadband Networks," *IEEE Journal on Selected Areas in Communications*, Vol.6, No.9, pp.1598-1608, December 1988.

[13] M.H. Ackroyd, "Computing the Waiting Time Distribution for the $G/G/1$ Queue by Signal Processing Methods," *IEEE Transactions on Communications*, Vol.COM-28, No.1, pp.52-58, January 1980.

Appendix: Determination of $X(z)$

To determine the unknown function $X(z)$, we first examine the relationship between $X(z)$ and $A(z)$, the z-transform for the interarrival time distribution of GI-stream cells. If $A(z)$ is a rational function of z, it is written as

$$A(z) = A_1(z) + A_2(z), \tag{A.1}$$

where $A_1(z)$ is a polynomial function:

$$A_1(z) = \sum_{i=1}^{M} m_i \, z^i, \tag{A.2}$$

and $A_2(z)$ is the ratio of two polynomial functions:

$$A_2(z) = \frac{\sum_{l=1}^{L} n_l \, z^l}{\prod_{h=1}^{H} (1 - \alpha_h z)^{\omega_h}}, \tag{A.3}$$

where

$$\sum_{h=1}^{H} \omega_h = L. \tag{A.4}$$

The quantities $1/\alpha_h$ are the zeros of denominator in eq.(A.3) and ω_h are the corresponding multiplicities. $a(n)$, the density for the interarrival times of GI-stream cells, can be also divided into two terms, $a_1(n)$ and $a_2(n)$, where

$$a_1(n) = \sum_{i=1}^{M} m_n, \qquad 1 \leq n \leq M, \tag{A.5}$$

$$a_2(n) = \sum_{i=1}^{H} \sum_{h=0}^{\omega_i - 1} r_{ih} \, n^h \, \alpha^n, \qquad n \geq 1, \tag{A.6}$$

where r_{ih} can be obtained by inverting $A(z)$.

$X(z) = \sum_{m=1}^{\infty} x(m) z^m$ is also written as a sum of two functions $X_1(z)$ and $X_2(z)$.

$$X(z) = X_1(z) + X_2(z). \tag{A.7}$$

The unknown sequence, $x(m)(m \geq 1)$, in $X(z)$ can be also written as a sum of two terms, $x_1(m)$ and $x_2(m)$, where

$$\begin{aligned} x_1(m) &= \sum_{k=1}^{\infty} a_1(m+k)\, C_k(0) \\ &= \sum_{k=1}^{\infty} C_k(0)\, m_{m+k} \\ &\triangleq x^\star(m), \quad 1 \leq m \leq M-1, \end{aligned} \tag{A.8}$$

and

$$\begin{aligned} x_2(m) &= \sum_{k=1}^{\infty} a_2(m+k)\, C_k(0) \\ &= \sum_{k=1}^{\infty} C_k(0) \sum_{i=1}^{H} \sum_{h=0}^{\omega_i - 1} r_{ih}(m+k)^{h-n}\, \alpha_i^{m+k} \\ &= \sum_{i=1}^{H} \sum_{n=0}^{\omega_i - 1} \left[\sum_{k=1}^{\infty} C_k(0)\, \alpha_i^k \sum_{h=n}^{\omega_i - 1} \binom{a}{b} k^{h-n} \right] m^n\, \alpha_i^m, \qquad m \geq 1. \end{aligned} \tag{A.9}$$

The z-transforms for $x_1(m)$ and $x_2(m)$ can be written as

$$X_1(z) = \sum_{i=1}^{M} x^\star(i)\, z^i, \tag{A.10}$$

$$X_2(z) = \frac{\sum_{l=1}^{L} x^{\star\star}(l)\, z^l}{\prod_{h=1}^{H} (1 - \alpha_h z)^{\omega_h}}. \tag{A.11}$$

$X_2(z)$ was derived by comparing eq.(A.9) and eq.(A.6). We now introduce the following functions:

$$P(z) = \sum_{i=1}^{M} m_i\, z^{M-i}\, B(z)^i, \tag{A.12}$$

$$Q(z) = \sum_{l=1}^{L} n_l\, z^{L-l}\, B(z)^l, \tag{A.13}$$

$$\Pi(z) = \prod_{h=1}^{H} (z - \alpha_h\, B(z))^{\omega_h}, \tag{A.14}$$

$$X^\star(z) = \sum_{i=1}^{M-1} x^\star(i)\, B(z)^i\, z^{M-i}, \tag{A.15}$$

$$X^{**}(z) = \sum_{l=1}^{L} x^{**}(l)\, B(z)^l\, z^{L-l}. \tag{A.16}$$

Using the above functions on eq.(A.1) and eq.(A.7), we can obtain the following equations:

$$A(\frac{B(z)}{z}) = \frac{\Pi(z)P(z) + z^M Q(z)}{z^M \Pi(z)}, \tag{A.17}$$

$$X(\frac{B(z)}{z}) = \frac{\Pi(z)X^{*}(z) + z^{M-1}X^{**}(z)}{z^{M-1}\Pi(z)}. \tag{A.18}$$

Substituting eqs.(A.17) and (A.18) into eq.(14), we finally obtain $C_0(z)$ in terms of $X^{*}(z)$ and $X^{**}(z)$ as

$$C_0(z) = \frac{(z-1)B(0) + \Pi(z)X^{*}(z) + z^{M-1}X^{**}(z)}{B(z)[z^{M-1}\Pi(z) - \Pi(z)P(z) + z^M Q(z)]}. \tag{A.19}$$

The denominator in the above equation has $M+L-1$ zeroes inside the unit disk of the complex plane whenever the stochastic equilibrium condition is fulfilled. However, since $C_0(z)$ is a z-transform, it cannot have any poles inside the unit disk. This implies that the $M+N-1$ zeroes of the denominator must be zeroes of the numerator of Eq.(A.19). Then, the unknown parameters $x_1(m)(1 \le m \le M-1)$ and $x_2(m)(1 \le m \le L)$ can be determined with $M+L-2$ linear equations with the normalizing condition $C_0(1) = 1$. This enables us to obtain $C_0(z)$ in Eq.(A.19) and $W_M(z)$ in Eq.(22).

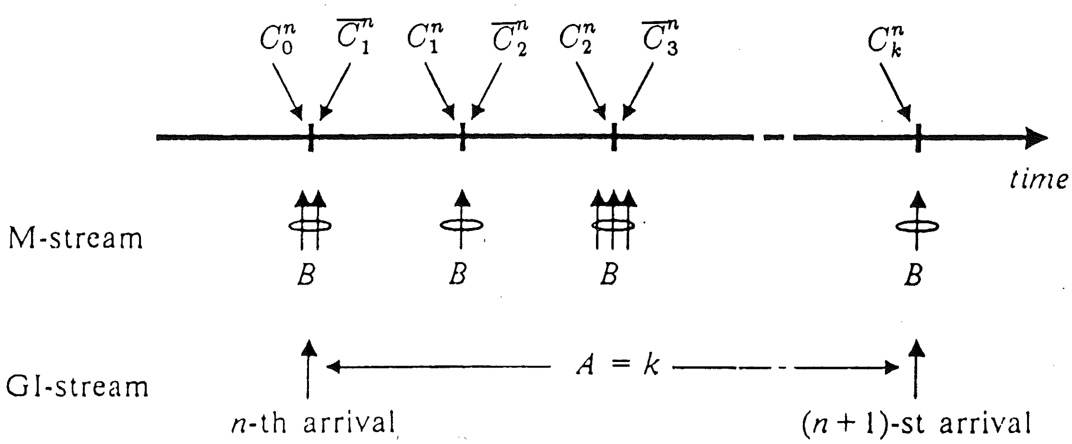

Fig.1 Sample Path for Random Variables

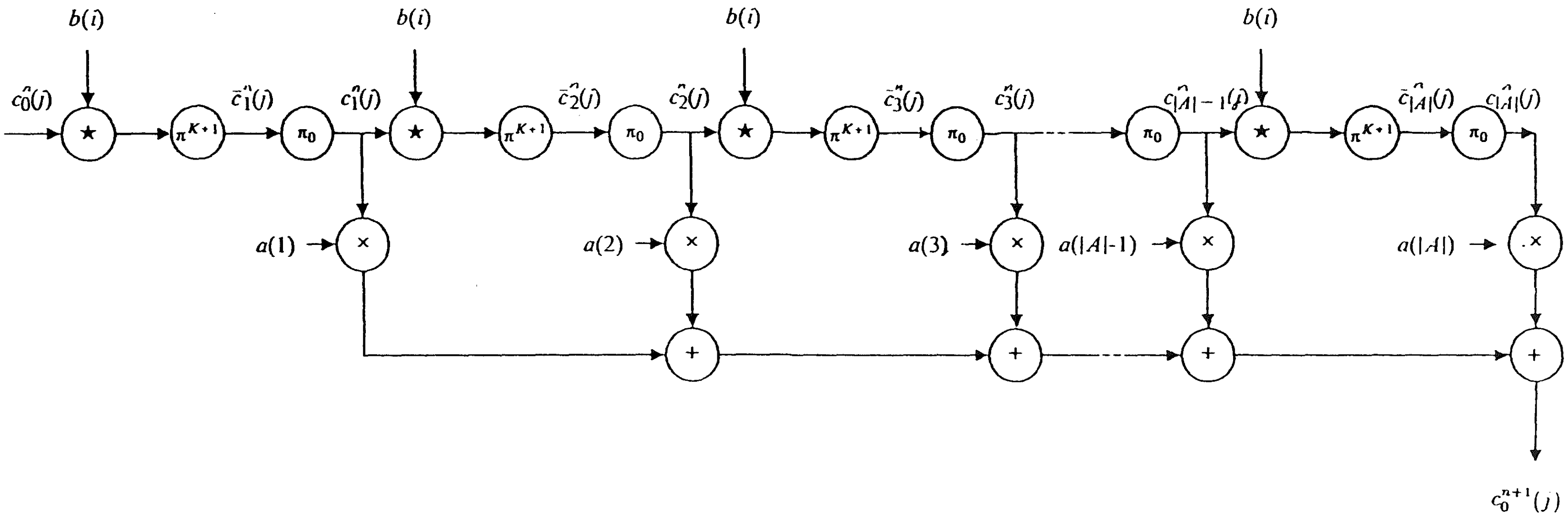

Fig.2 Computational Diagram for Computing Buffer Length Distributions

($|A|$ represents the maximum interarrival time for the density $a(k)$)

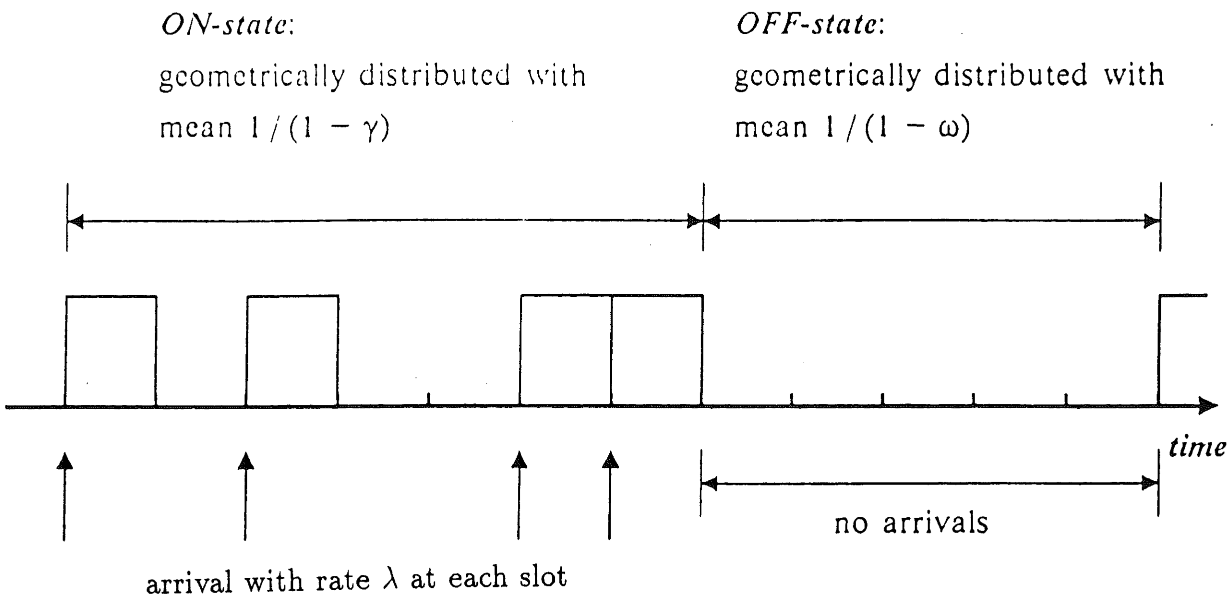

Fig.3 Discrete-Time Interrupted Poisson

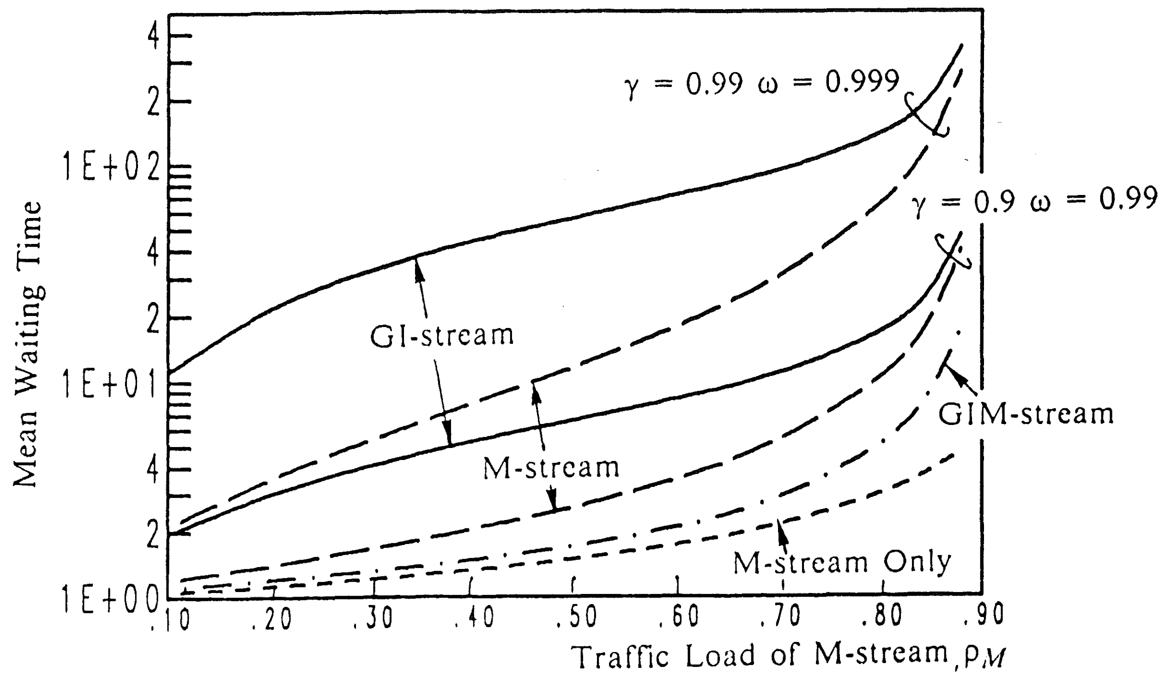

Fig.4.1 Mean Waiting Times (Infinite Buffer System)

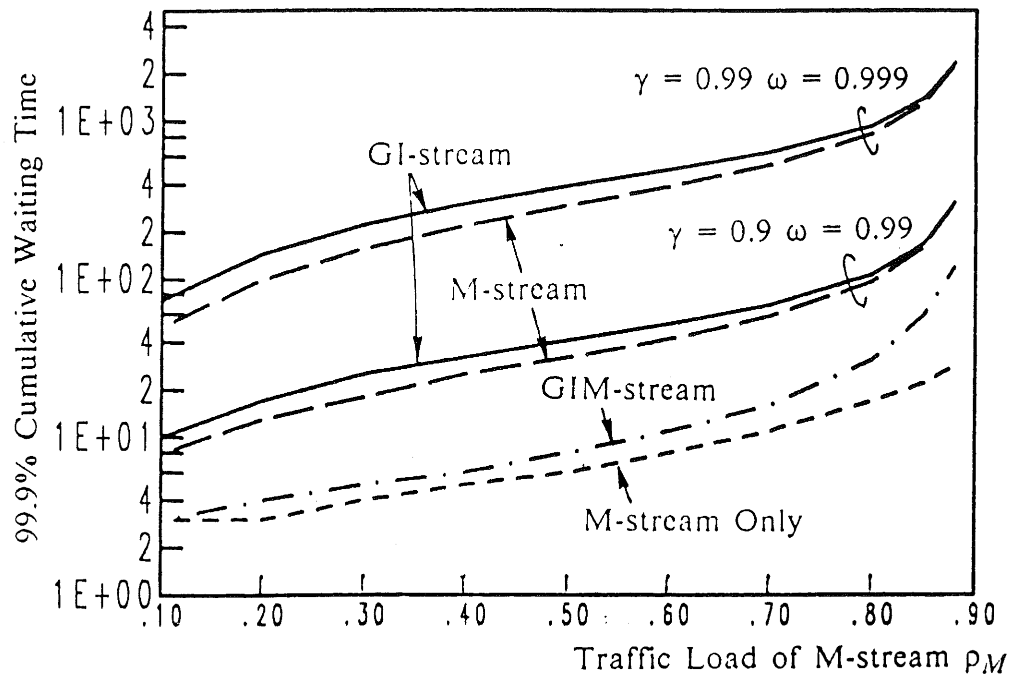

Fig.4.2 99.9% Cumulative Waiting Times (Infinite Buffer System)

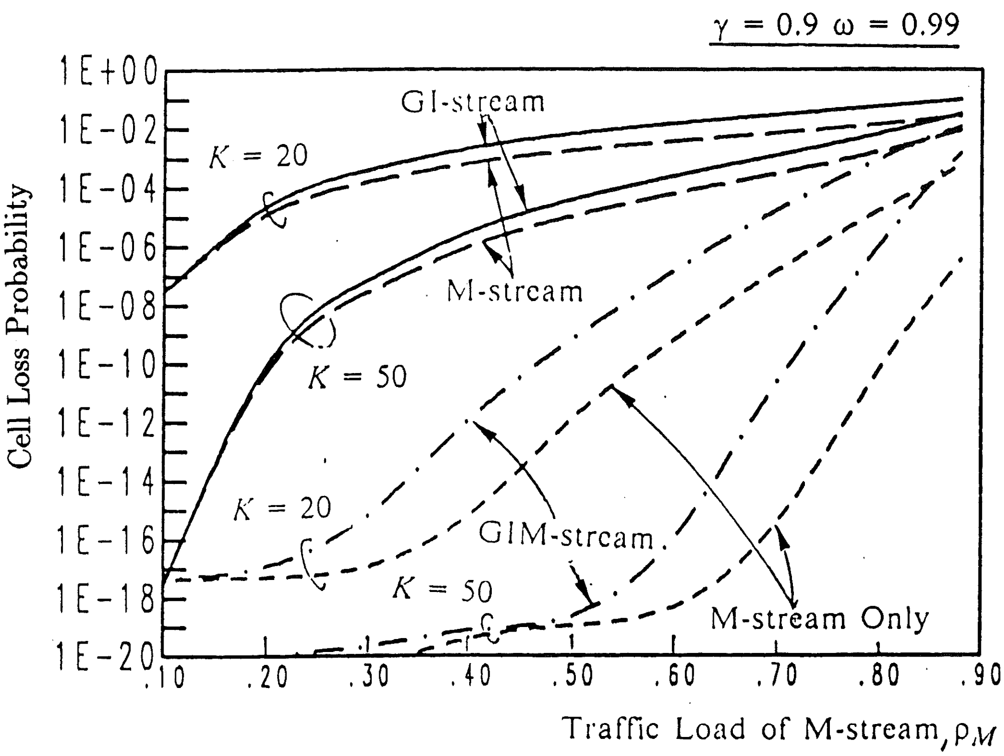

Fig.4.3 Cell Loss Probability (Finite Buffer System)

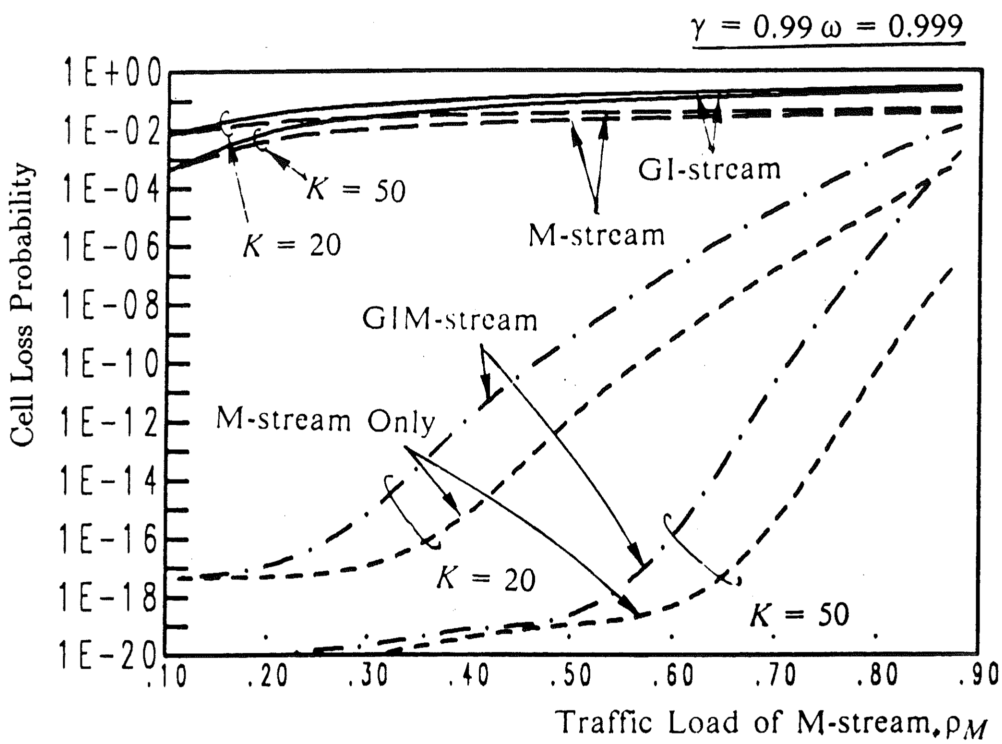

Fig.4.4. Cell Loss Probability (Finite Buffer System)

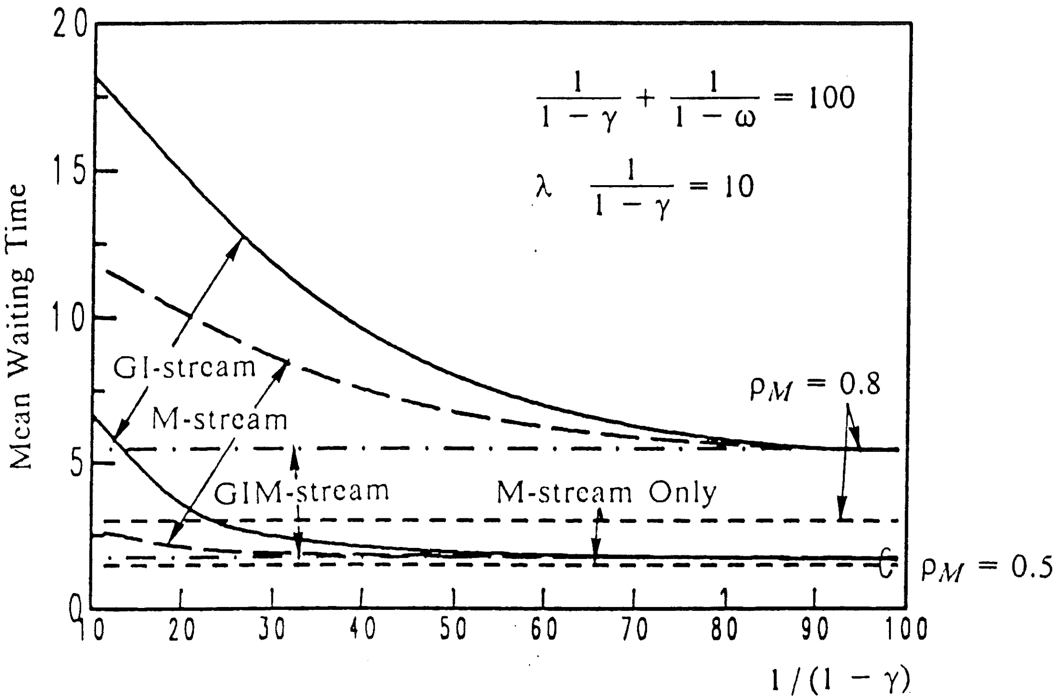

Fig.5.1 Effect of Traffic Smoothing on Mean Waiting Times

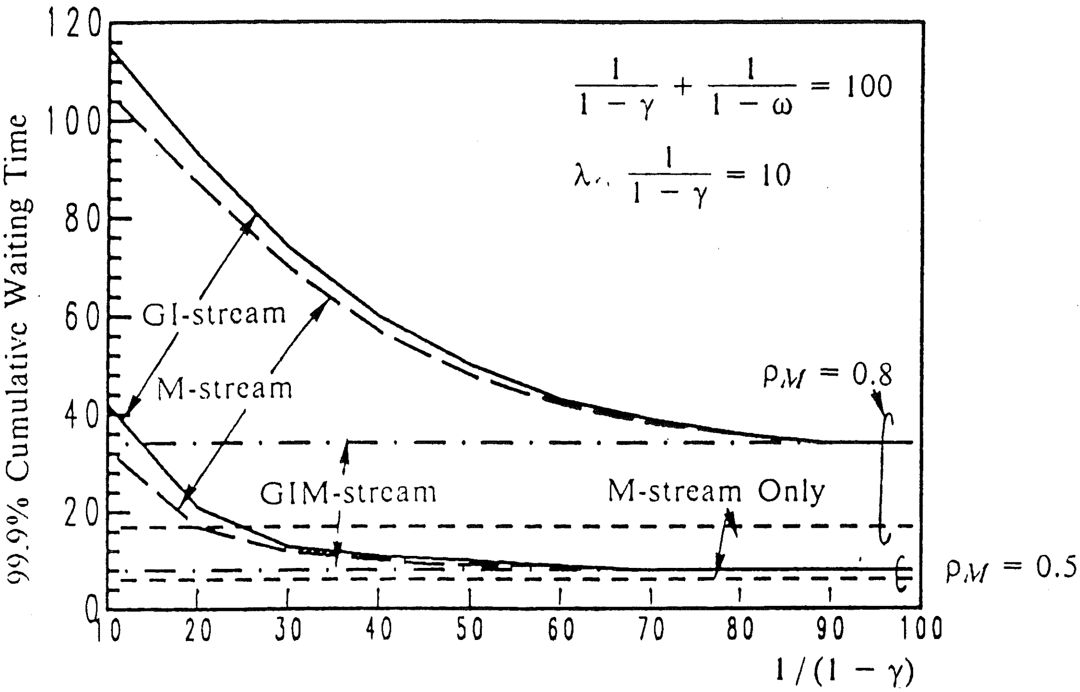

Fig.5.2 Effect of Traffic Smoothing on 99.9% Cumulative Waiting Times

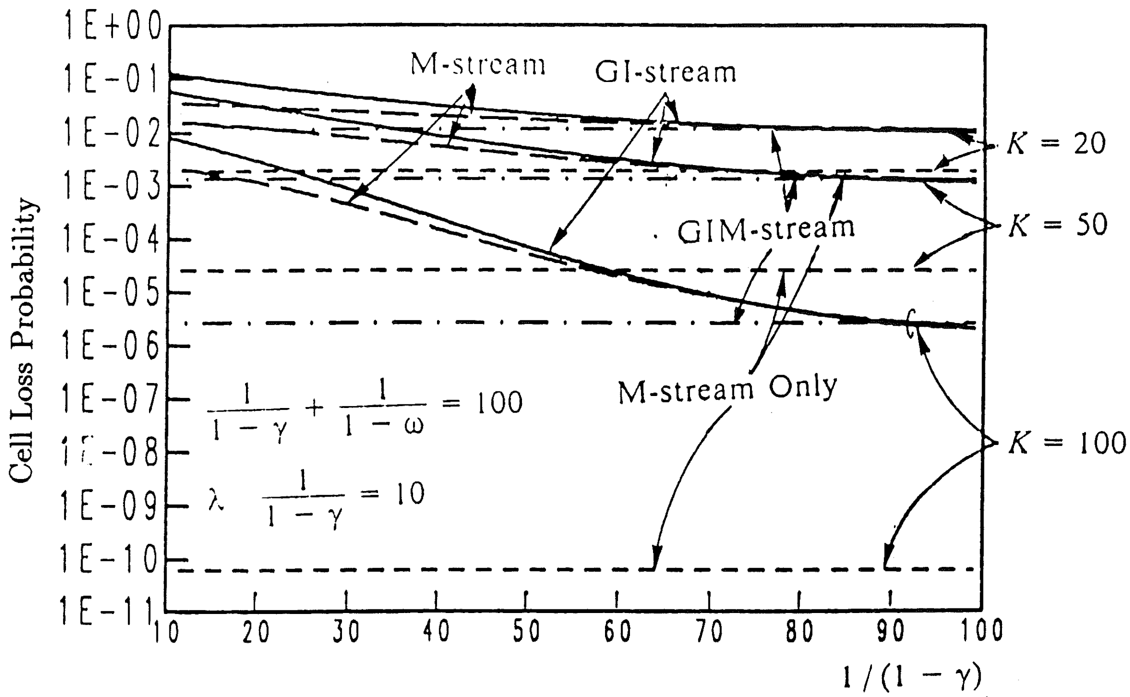

Fig.5.3 Effect of Traffic Smoothing on Cell Loss Probability

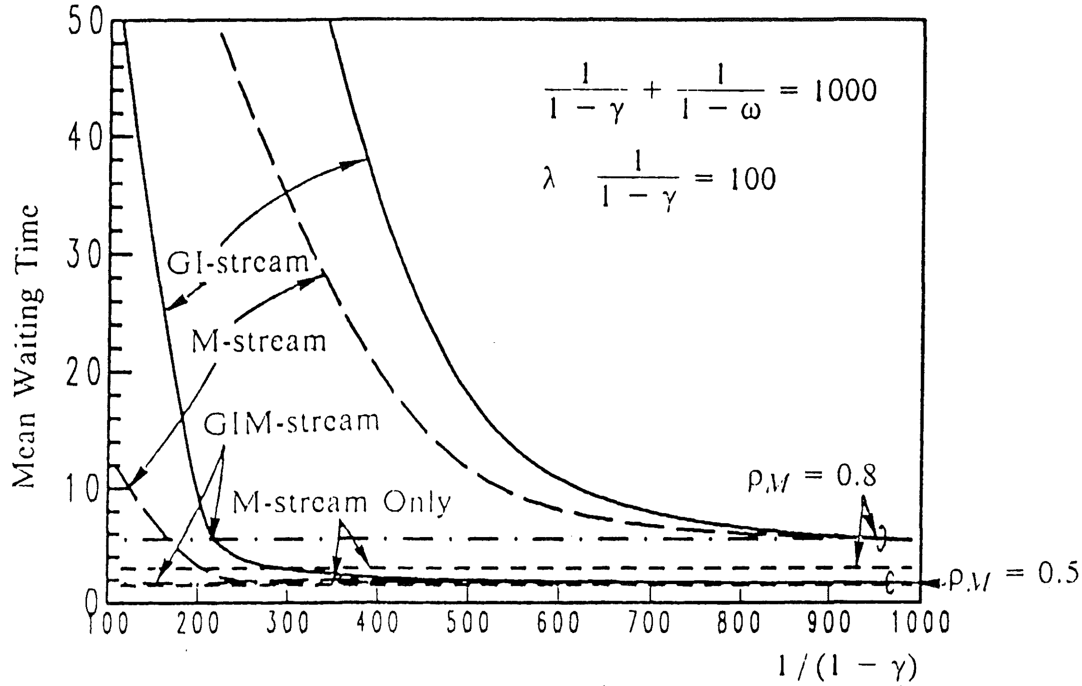

Fig.5.4 Effect of Traffic Smoothing on Mean Waiting Times